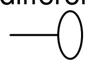
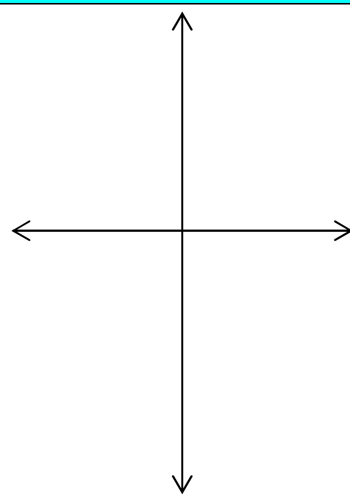
6.1.2: Graphing Quadratic Equations

1. Obtain a pair of equations from your teacher.
2. Press the **Zoom** button and press **6** (for ZStandard) to set the window to make the max and min on both axes go from –10 to 10.
3. Press the ***Y* =** button and key in your two equations into Y_1 and Y_2.
4. To change the graph of Y_2 to "animation": Move the cursor to the left of Y_2. Press **Enter** four times to toggle through different graph styles available.
   You should see 
5. Press **Graph**. First the Y_1 quadratic will appear, then the Y_2 quadratic will appear and be traced by an open circle.
6. Complete the three columns of the table below.

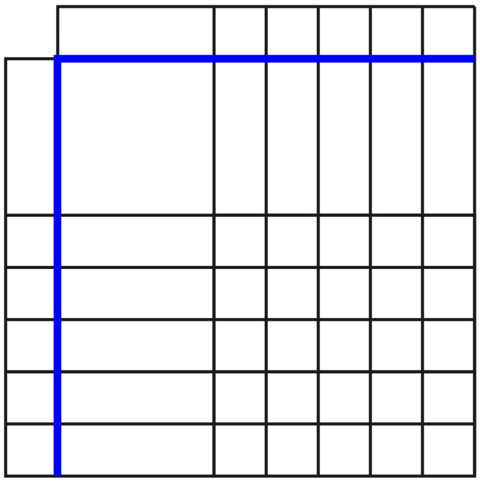
Our Two Equations	What They Look Like	What We Think It Means
		

7. Press 2[nd] **Graph** so that you can look at the tables of values for the two curves. Discuss what you see and complete the table.

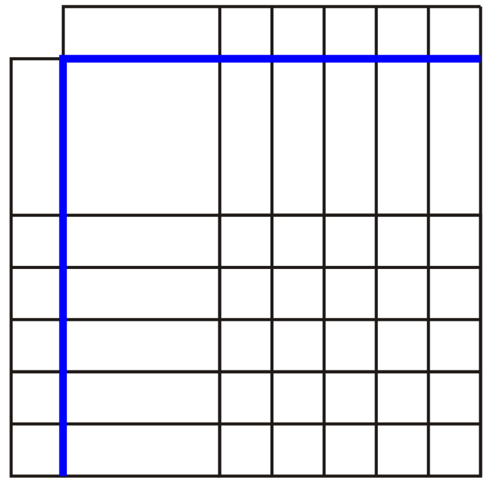
What We Noticed About the Table of Values	What We Think It Means

6.1.3: Algebra Tile Template

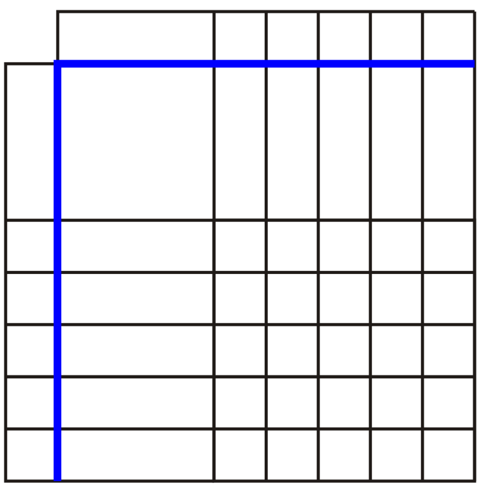
1. $y = (\qquad)(\qquad) =$ ________________



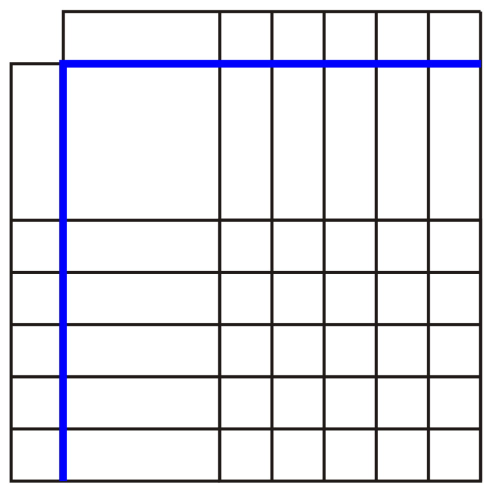
2. $y = (\qquad)(\qquad) =$ ________________



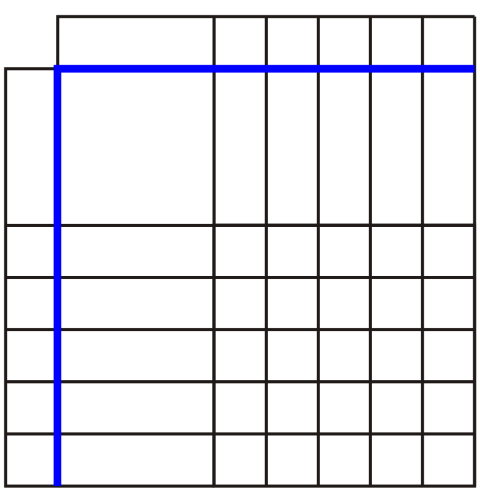
3. $y = (\qquad)(\qquad) =$ ________________



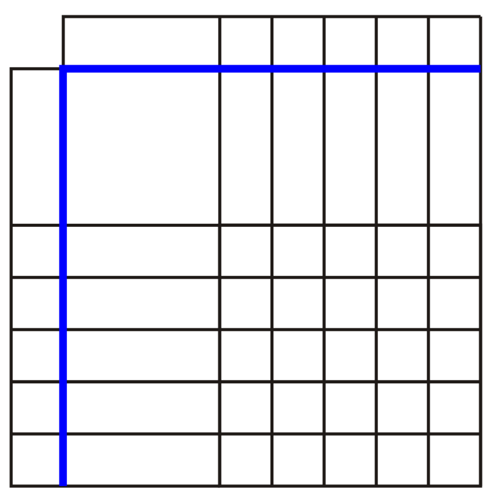
4. $y = (\qquad)(\qquad) =$ ________________



5. $y = (\qquad)(\qquad) =$ ________________



6. $y = (\qquad)(\qquad) =$ ________________



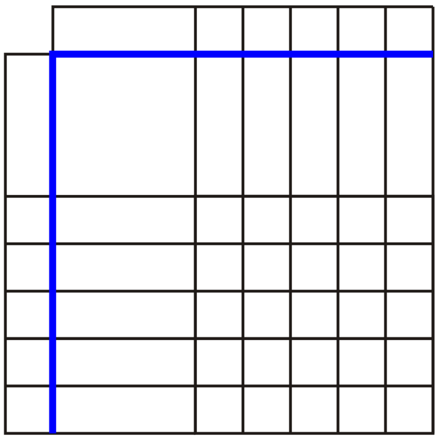
6.2.1: Multiply a Binomial by a Binomial

Name:

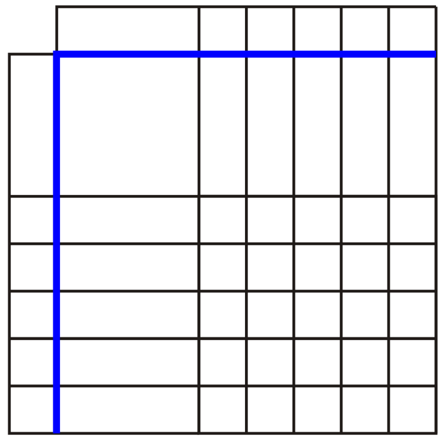
Part A

Use algebra tiles to multiply binomials and simplify the following:

1. $y = (x + 1)(x + 3) =$ ____________________



2. $y = (x + 2)(x + 3) =$ ____________________



Part B

Use the chart method to multiply and simplify the following:

1. $y = (x + 1)(x + 3) =$ ____________________

2. $y = (x + 2)(x +3) =$ ____________________

3. $y = (x + 2)(x - 1) =$ ____________________

4. $y = (x - 2)(x + 3) =$ ____________________

5. $y = (x - 1)(x - 1) =$ ____________________

6. $y = (x - 1)\ (x - 2) =$ ____________________

6.2.1: Multiply a Binomial by a Binomial (continued)

Part C

Multiply and simplify the two binomials, using the chart method and the distributive property.

1. $(x + 4)(x - 3)$

	x	$+4$
x		
-3		

2. $(x - 3)(x - 3)$

	x	-3
x		
-3		

3. $(x + 2)^2$

	x	$+2$
x		
$+2$		

4. $(x + 2)(x - 1)$

	x	$+2$
x		
-1		

5. $(x - 2)\ (x + 1)$

	x	-2
x		
$+1$		

6. $(x - 1)^2$

	x	-1
x		
-1		

7. $(x - 1)(x - 2)$

	x	-1
x		
-2		

8. $(x - 3)(x - 4)$

	x	-3
x		
-4		

6.2.2: Chart Template for Distributive Property

1. y = (   )(   ) = ______________

2. y = (   )(   ) = ______________

3. y = (   )(   ) = ______________

4. y = (   )(   ) = ______________

5. y = (   )(   ) = ______________

6. y = (   )(   ) = ______________

7. y = (   )(   ) = ______________

8. y = (   )(   ) = ______________

6.3.1: Finding the *y*-Intercept of a Quadratic Equation

1. Use the graphing calculator to find the *y*-intercept for each of the equations:

   Note any patterns you see.

Equation	*y*-intercept
$y = x^2 - x - 2$	
$y = x^2 + 2x - 8$	
$y = x^2 - x + 6$	
$y = (x-1)(x-2)$	
$y = (x+4)(x+3)$	
$y = (x+3)^2$	

2. How can you determine the *y*-intercept by looking at a quadratic equation?

3. Which form of the quadratic equation is easiest to use to determine the *y*-intercept? Explain your choice.

4. Using your conclusion from question 2, state the *y*-intercept of each and check using a graphing calculator.

Equation	*y*-intercept	Does it check?	
		Yes	No
$y = x^2 - 2x - 8$			
$y = x^2 - x - 6$			
$y = x^2 + 3x + 2$			
$y = (x-4)(x-1)$			
$y = (x-2)(x+5)$			

5. Explain the connection between the *y*-intercept and the value of *y* when $x = 0$.

6.3.2: Quadratic Equations

1. Find the *y*-intercept for each of the following quadratic equations given in factored form. Write the equations in standard form. Show your work.

   a) $y = (x-5)(x+2)$
   $=$
   $=$

   standard form:

   y-intercept:

   b) $y = (x+4)(x-3)$
   $=$
   $=$

   standard form:

   y-intercept:

   c) $y = (x-4)^2$
   $=$
   $=$

   standard form:

   y-intercept:

   d) $y = (x+5)^2$
   $=$
   $=$

   standard form:

   y-intercept:

2. Find the *y*-intercept for each of the following quadratic equations:

   a) $y = (x+4)(x+2)$

   y-intercept:

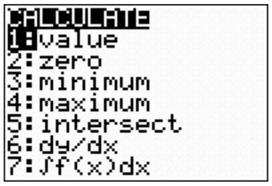
   b) $y = (x-6)^2$

   y-intercept:

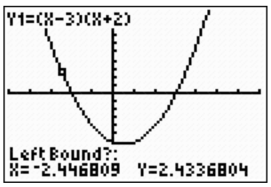
6.4.1: Finding the *x*-Intercepts of a Quadratic Equation

To find the *x*-intercepts:

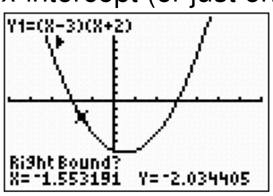
1. Enter the equation in Y_1. $y = x^2 - x - 6$
2. Press **ZOOM** and **6** (Zstandard) to set the scale for your graph. The calculator will then show the parabola.
3. Press **2nd TRACE** 1 to view the Calculate screen.



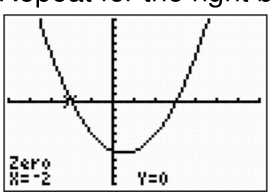
4. Select **2: ZERO**. Your screen should be similar to the following screen.



5. You will be asked to enter a left bound. You can move the cursor to the left of one *x*-intercept (or just enter an *x* value that is to the left of the *x*-intercept). Press **ENTER.**



6. Repeat for the right bound, being sure that you are to the right of the same *x*-intercept.



7. The next screen will say guess. You can guess if you want but it is not necessary. Press **ENTER.** You will get one *x*-intercept.

8. Repeat steps 3 through 7 to get the other *x*-intercept.

6.4.1: Finding the *x*-Intercepts of a Quadratic Equation (continued)

1. Use the graphing calculator to find the *x*-intercepts for each of the following:

Equation	First *x*-intercept	Second *x*-intercept
$y = x^2 - 4x - 12$		
$y = x^2 + 2x - 8$		
$y = x^2 - x + 6$		
$y = (x-1)(x-2)$		
$y = (x+4)(x+3)$		
$y = (x-3)(x+5)$		

2. Can you determine the *x*-intercepts by looking at a quadratic equation? Explain.

3. Which form of the quadratic equation did you find the easiest to use when determining the *x*-intercepts? Explain the connection between the factors and the *x*-intercepts.

6.4.2: Area with Algebra Tiles

Using algebra tiles create the rectangles for the following areas.
Complete the following chart.

Area of Rectangle	Number of x^2 Tiles	Number of x Tiles	Number of Unit Tiles	Sketch of Rectangle	Length	Width
$x^2 + 4x + 3$						
$x^2 + 5x + 6$						
$x^2 + 6x + 8$						
$x^2 + 7x + 12$						

1. Find a relationship between the number of x tiles and the numbers in the expressions for the length and width.

2. Find a relationship between the number of unit tiles and the numbers in the expressions for the length and width.

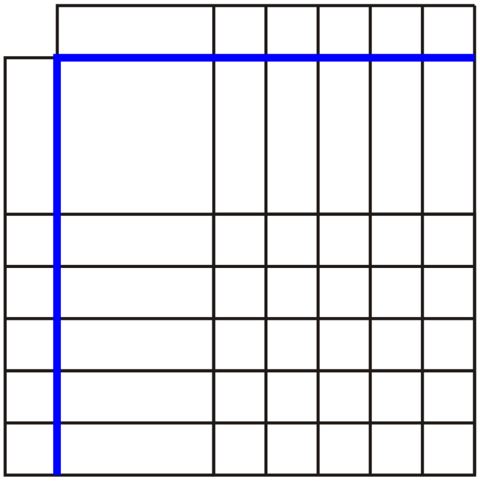
3. If the area of a rectangle is given by $x^2 + 8x + 15$, what expression will represent the length and the width?

6.4.3: Factoring Using Algebra Tiles

Part A

For each of the following, shade in the appropriate rectangular area. Then shade in the tiles that represent the length and width for each of those areas. Use the length and width to represent and state the factors. State the *x*-intercepts. Check using a graphing calculator.

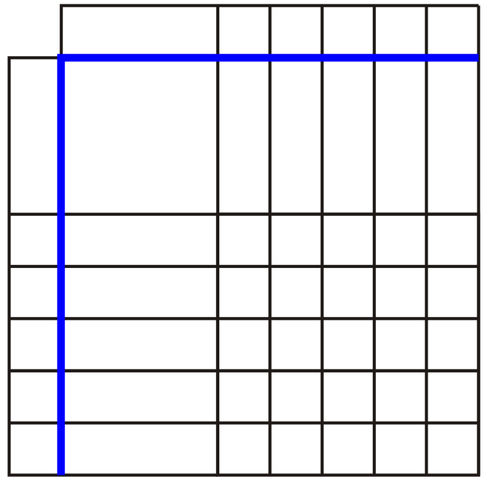
1. $y = x^2 + 3x + 2 = (\qquad)(\qquad)$



x-intercepts ______, ______

Check with calculator.

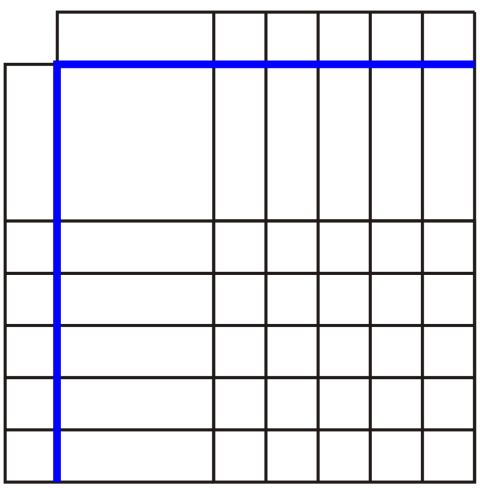
2. $y = x^2 + 5x + 4 = (\qquad)(\qquad)$



x-intercepts ______, ______

Check with calculator.

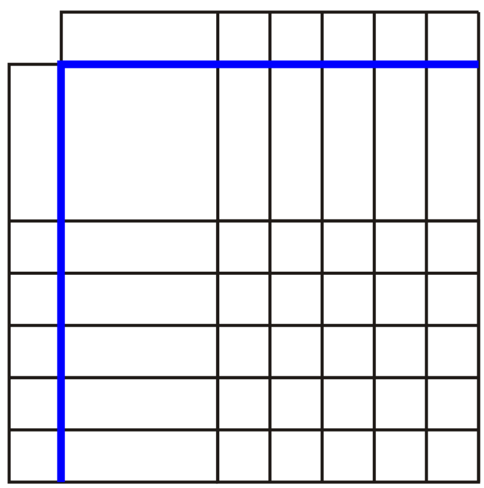
3. $y = x^2 + 6x + 5 = (\qquad)(\qquad)$



x-intercepts ______, ______

Check with calculator.

4. $y = x^2 + 4x + 4 = (\qquad)(\qquad)$



x-intercepts ______, ______

Check with calculator.

6.4.3: Factoring Using Algebra Tiles (continued)

Part B

Using the diagrams in Part A, find the x- and y-intercepts for each quadratic relation. Use the information to make the sketch on the grid provided.

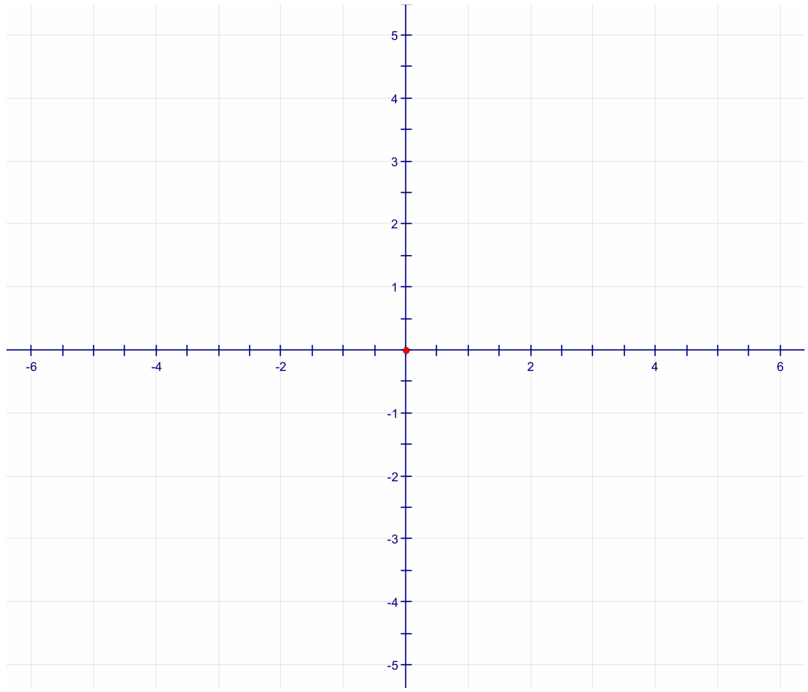
1. standard form:
   $y = x^2 + 3x + 2$

   factored form:

   y-intercept:

   first x-intercept:

   second x-intercept:



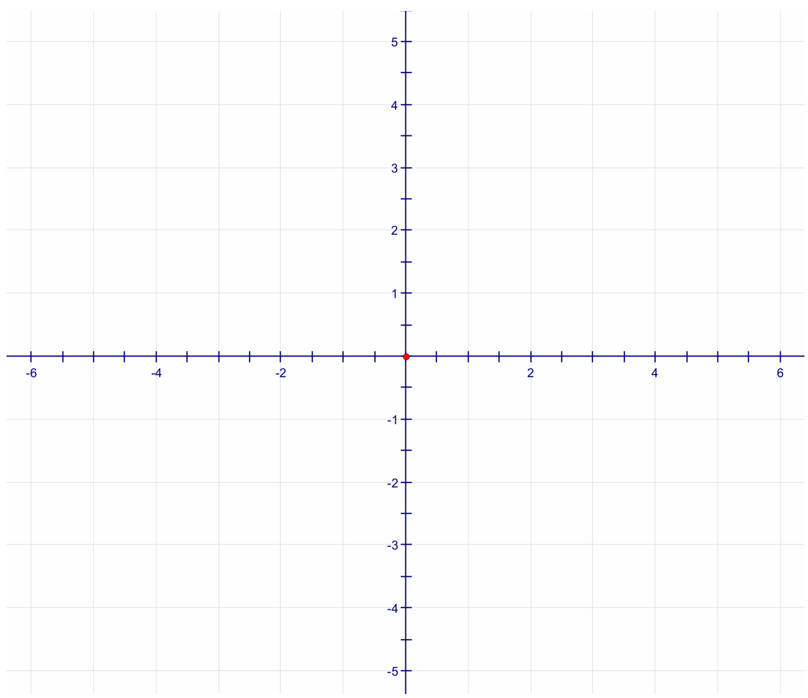
2. standard form:
   $y = x^2 + 5x + 4$

   factored form:

   y-intercept:

   first x-intercept:

   second x-intercept:



6.4.3: Factoring Using Algebra Tiles (continued)

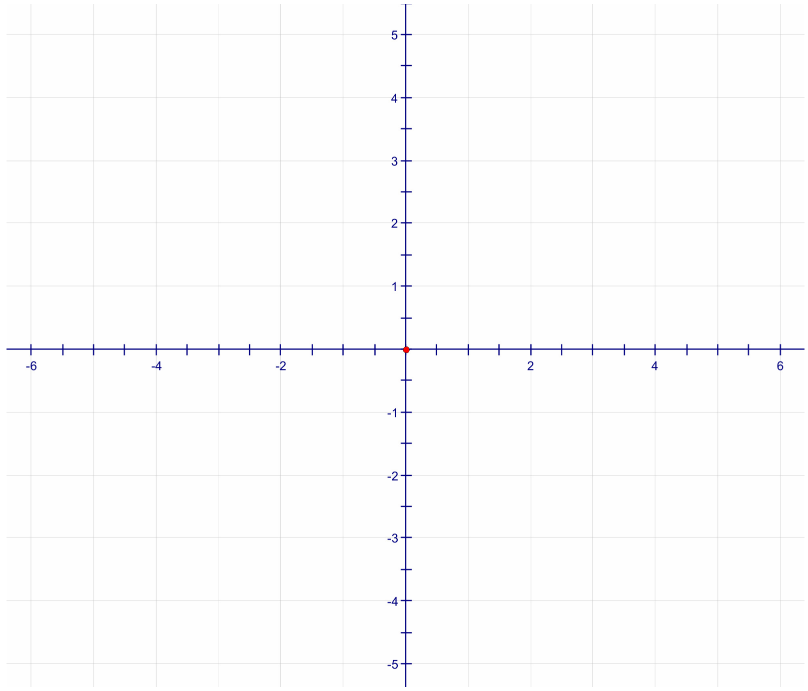
3. standard form:
   $y = x^2 + 6x + 5$

   factored form:

   y-intercept:

   first x-intercept:

   second x-intercept:



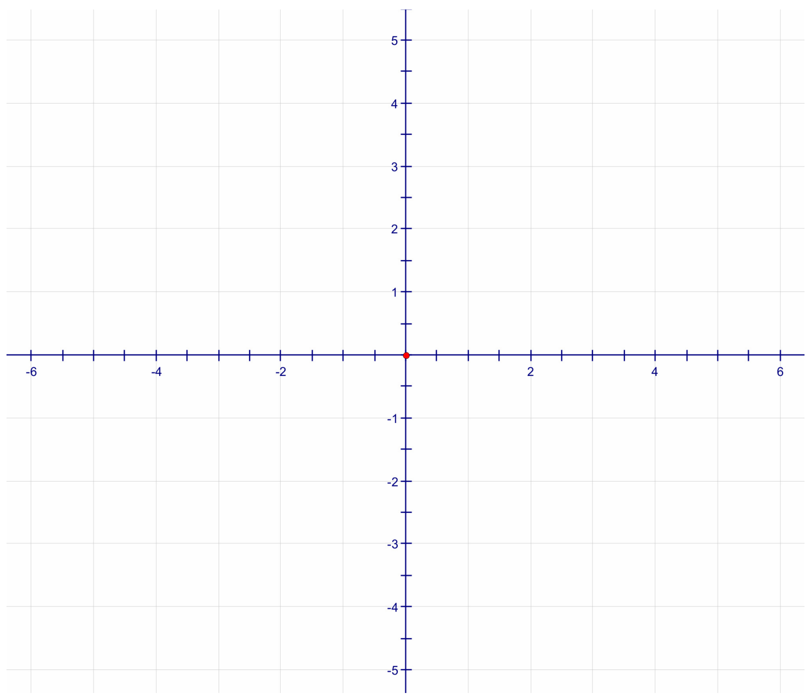
4. standard form:
   $y = x^2 + 4x + 4$

   factored form:

   y-intercept:

   first x-intercept:

   second x-intercept:



5. In what way is the last example different from the others?

6.5.2: Factored Form and *x*-Intercepts

Use algebra tiles to find the length and width for each given area. Use the graphing calculator to find the *x*-intercepts of the corresponding quadratic relation. Graph both the area model and factored form of the quadratic relation to check that these are the same before finding the *x*-intercepts.

Area	Length	Width	Factored Form	First *x*-intercept of corresponding relation	Second *x*-intercept of corresponding relation
$x^2 + 4x + 3$					
$x^2 + 5x + 6$					
$x^2 + 6x + 8$					
$x^2 + 7x + 12$					

1. What do you notice about the constant term in the length and width expressions and the coefficient of the x term in the area expressions?

2. What do you notice about the constant term in the length and width expressions and the constant term in the area expressions?

3. If an area is expressed as $x^2 + 10x + 21$, what must be true of the constant terms in the length and width expressions?

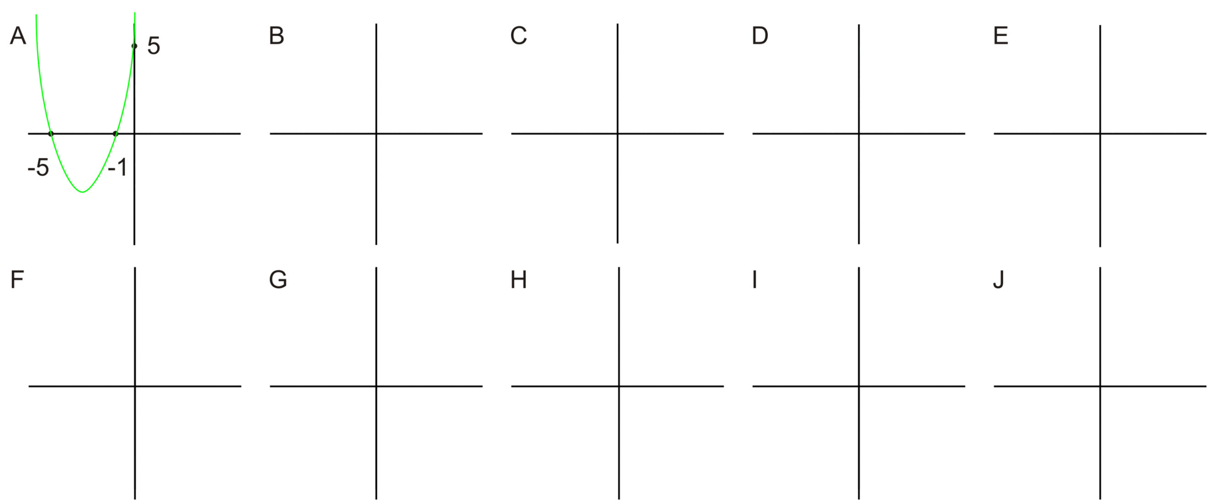
4. If the standard form of a quadratic relation is $y = x^2 + bx + c$, and it has x-intercepts of r and s, then the same relation would then be $y = (x - r)(x - s)$. How would you find the value of r and s?

6.6.1: Use Intercepts to Graph It!

Given the standard form of the quadratic relation, identify the value of the sum and product needed to factor. Express the relation in factored form, identify the *x*-intercepts and *y*-intercept, and use these results to make a sketch of each parabola.

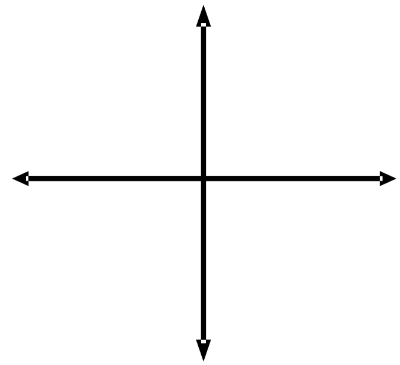
	Standard Form	**Factored Form**	***x*-intercepts**	***y*-intercept**
A	$y = x^2 + 6x + 5$	$y = (x+1)(x+5)$	–1 and –5	5
B	$y = x^2 - 4x - 5$			
C	$y = x^2 + 4x - 5$			
D	$y = x^2 - 6x + 5$			
E	$y = x^2 + 7x + 6$			
F	$y = x^2 - 6x + 9$			
G	$y = x^2 - x - 6$			
H	$y = x^2 + 13x + 12$			
I	$y = x^2 - 4x - 12$			
J	$y = x^2 + x - 12$			

Sketch each relation (considering x-intercepts and y-intercept)

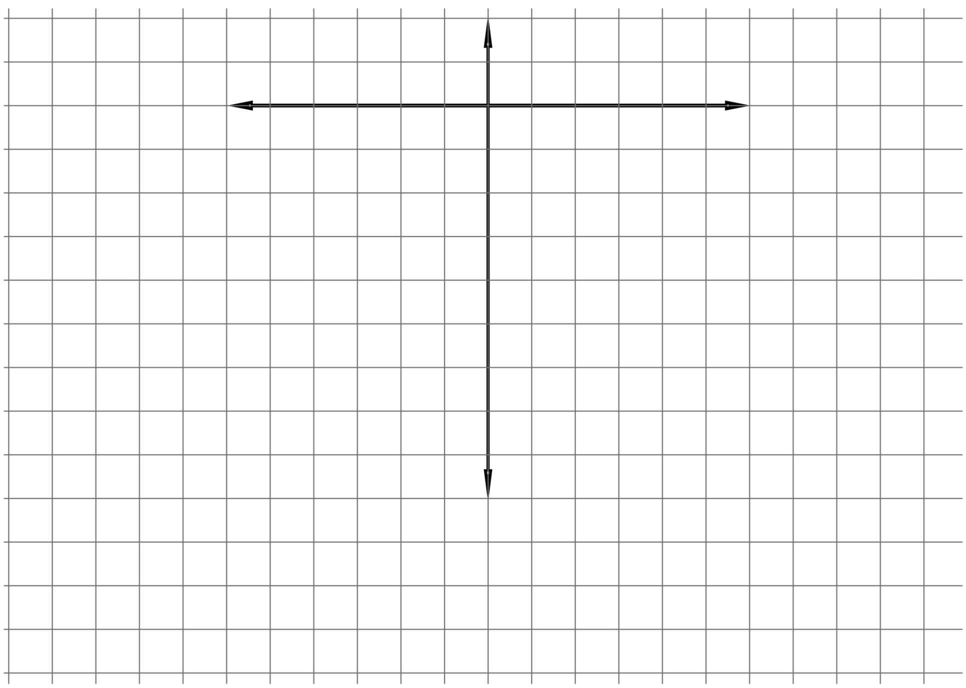


6.7.1: Investigate Relations of the Form $y = ax^2 + b$

1. Obtain a graphing calculator and equation from your teacher.
2. Type in the equation (using the **Y =** button on your calculator). Key in **zoom 6** to get the max and min from –10 to 10 on your window.
3. Fill in the table.

Your equation	**A sketch of your quadratic equation (including the *x*- and *y*-intercepts)**
$y =$ Coordinates of *x*-intercepts (______, ______) and (______, ______) and the coordinate of the *y*-intercept (______, ______)	

4. In your group, sketch all four graphs.



5. Identify what is the same and what is different in these four graphs.

6.7.1: Investigate Relations of the Form $y = ax^2 + b$ (continued)

6. Fill in the following table.

Standard form	Write each equation in factored form $y = (x - r)(x - s)$
$y = x^2 + 4x$	
$y = x^2 + 6x$	
$y = x^2 - 2x$	
$y = x^2 - 5x$	

7. Clear the y = screen on one of the four calculators, enter the factored form of the four equations, and graph. Are these graphs the same as the ones in your sketch?
   If yes, continue to question 8. If no, revise and check. Ask your teacher for assistance, if needed.

8. Can the equations in the third column of your table be simplified? Explain.

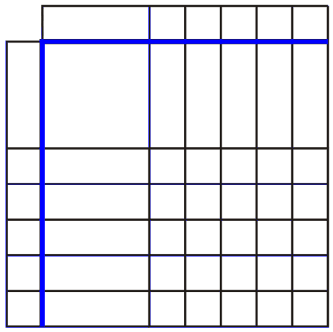
9. Record the simplified versions of your relation in factored form.

Standard Form	Factored Form
$y = x^2 + 4x$	
$y = x^2 + 6x$	
$y = x^2 - 2x$	
$y = x^2 - 5x$	

6.7.2: Factoring $x^2 + bx$

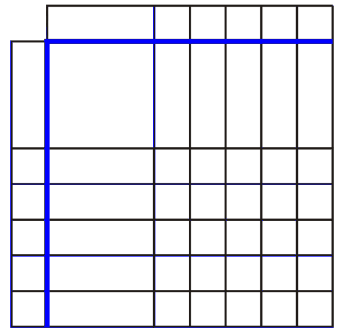
Consider the outer portion of the algebra tile representation as the length and width of a room. The rectangle is the carpet. Colour in as many cells as required for each example to form a rectangle. To factor, form a rectangle using the tiles, then determine the length and width of the room.

Example 1: $x^2 + 2x$



factored form (_______) (_______)
the coordinates of the x-intercepts of $y = x^2 + 2x$
(_______, _______) and (_______, _______)
the coordinate of the y-intercept (_______, _______)

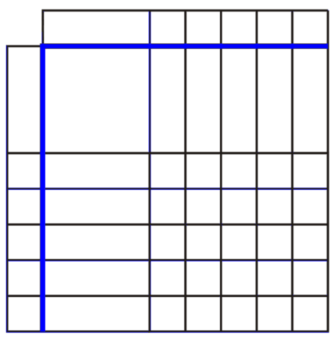
Example 2: $x^2 + 3x$



factored form (_______) (_______)
the coordinates of the x-intercepts of $y = x^2 + 3x$
(_______, _______) and (_______, _______)
the coordinate of the y-intercept (_______, _______)

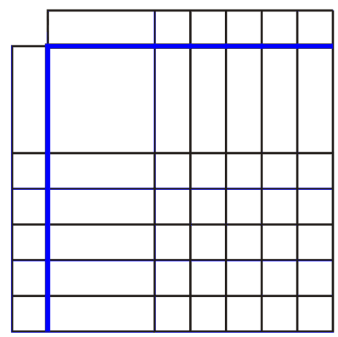
Use your algebra tiles to factor the following:

1. $x^2 + 4x$



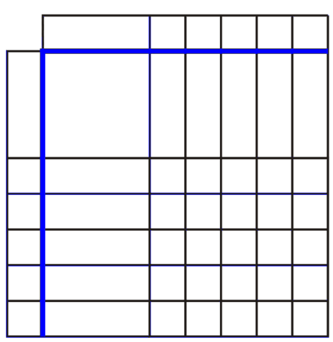
factored form (_______) (_______)
the coordinates of the x-intercepts of $y = x^2 + 4x$
(_______, _______) and (_______, _______)
the coordinate of the y-intercept (_______, _______)

2. $x^2 + 1x$



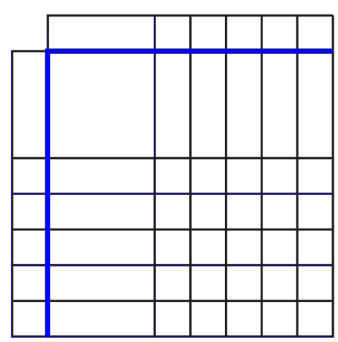
factored form (_______) (_______)
the coordinates of the x-intercepts of $y = x^2 + 1x$
(_______, _______) and (_______, _______)
the coordinate of the y-intercept (_______, _______)

3. $x^2 + 5x$



factored form (_______) (_______)
the coordinates of the x-intercepts of $y = x^2 + 5y$
(_______, _______) and (_______, _______)
the coordinate of the y-intercept (_______, _______)

4. $x^2 + x$



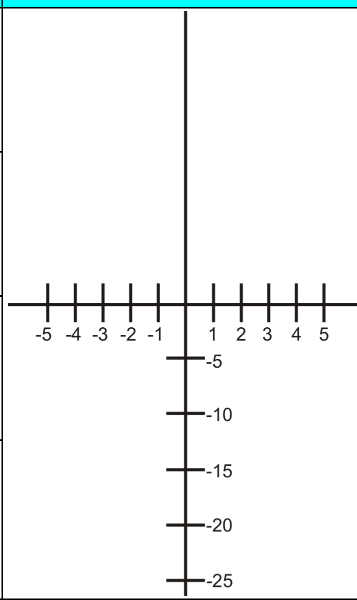
factored form (_______) (_______)
the coordinates of the x-intercepts of $y = x^2 + x$
(_______, _______) and (_______, _______)
the coordinate of the y-intercept (_______, _______)

6.8.1: Graphing Relationships of the Form $y = x^2 - a^2$

Name:

1. Working with a partner and one graphing calculator, set your Window: Xmin = –10; Xmax = 10; Xscl = 1; Ymin = –36; Ymax = 10; Yscl = 1; Xres = 1.

   Complete the following table.

<table>
<tr><th></th><th>Relation in standard form</th><th>Sketch each graph. Label as A, B, C, D, or use different colours.</th><th>y-intercept</th><th>x-intercept (r)</th><th>x-intercept (s)</th><th>Relation in factored form $y = (x - r)(x - s)$</th></tr>
<tr><td>A</td><td>$y = x^2 - 4$</td><td rowspan="4">
</td><td></td><td></td><td></td><td></td></tr>
<tr><td>B</td><td>$y = x^2 - 9$</td><td></td><td></td><td></td><td></td></tr>
<tr><td>C</td><td>$y = x^2 - 16$</td><td></td><td></td><td></td><td></td></tr>
<tr><td>D</td><td>$y = x^2 - 25$</td><td></td><td></td><td></td><td></td></tr>
</table>

Consider your results from question 1 and answer the following questions.

2. What is the same about the relations?

3. What is the same about the graphs?

4. What is the same about the vertex of each graph?

5. What do you notice about the r and s values of each relation?

6. Solve this puzzle. How can you find the y-intercept and the x-intercepts of the graph of a quadratic relation of the form $y = x^2 - a^2$?

6.9.1 Quick Review of Factoring and Graphing

For each of the following:

- identify what information the equation tells you about the parabola
- factor the equation and identify what the new form of the equation tells you
- sketch the parabola using the information you have (make sure you plot key points!)

Equation (Standard Form)	**Y-Intercept**	**Factored Form**	**Zeros**	**Sketched Graph**
1. $y = x^2-7x+10$				
2. $y = x^2 – x – 6$				

Equation (Factored Form)	Zeros	Standard Form	Y-Intercept	Sketched Graph
3. $y = (x – 2)(x – 5)$				
4. $y = (x-3)(x+2)$				

6.9.2 Matching

- For each equation in column A, state the y-intercept.
- For each equation in column B, state the zeros (or x-intercepts)
- Each equation in column A has a matching equation in column B. Draw an arrow

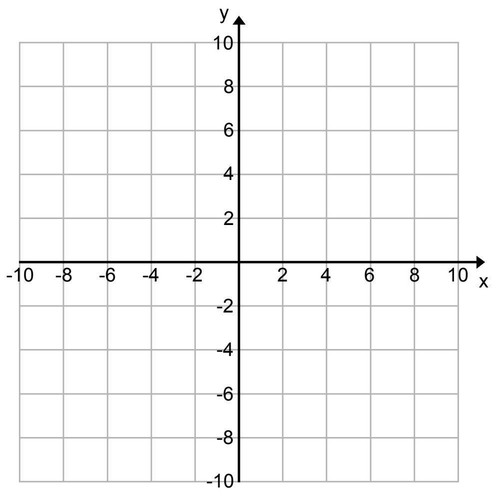
Column A	Column B
$y = x^2 + 3x + 2$ y-intercept = _______	$y = (x - 3)(x + 2)$ x-intercepts = _________________
$y = x^2 - 3x - 10$ y-intercept = _________	$y = (x - 3)(x - 2)$ x-intercepts = _________________
$y = x^2 + x - 12$ y-intercept = _________	$y = (x + 4)(x + 2)$ x-intercepts = _________________
$y = x^2 - 5x + 6$ y-intercept = __________	$y = (x + 2)(x + 1)$ x-intercepts = _________________
$y = x^2 + 6x + 8$ y-intercept = __________	$y = (x - 3)(x + 4)$ x-intercepts = _________________
$y = x^2 - 2x - 8$ y-intercept = __________	$y = (x + 5)(x + 2)$ x-intercepts = _________________
$y = x^2 + 3x - 4$ y-intercept = __________	$y = (x - 5)(x + 2)$ x-intercepts = _________________
$y = x^2 - x - 6$ y-intercept = ___________	$y = (x - 4)(x + 2)$ x-intercepts = _________________
$y = x^2 + 7x + 10$ y-intercept = ___________	$y = (x + 4)(x - 1)$ x-intercepts = _________________

6.9.3 Quick Sketches of Parabolas

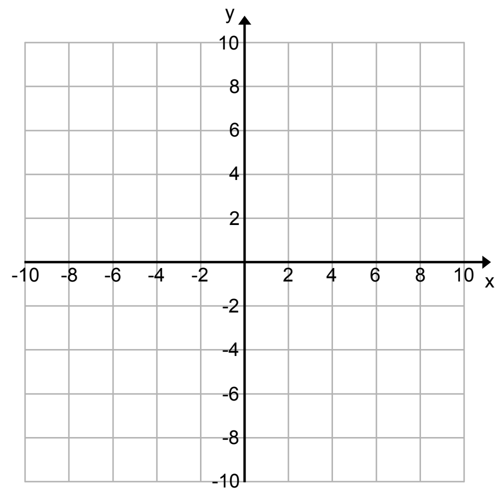
For each quadratic function given below determine:
• The zeros (*x*-intercepts)
• The *y*-intercept

With the information you find and using symmetry, graph each parabola.

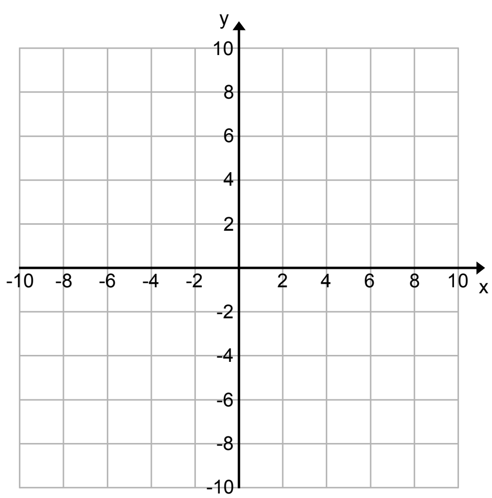
a) $y = x^2 + 6x + 8$



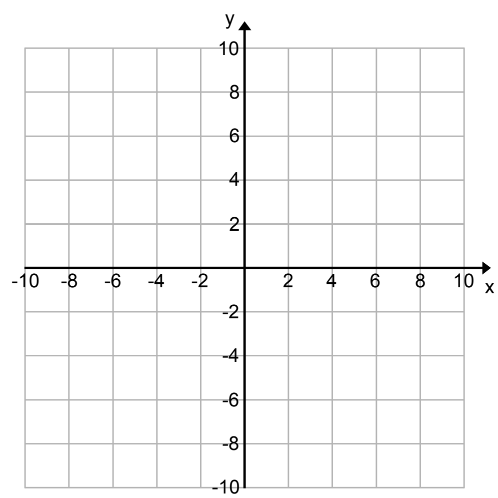
b) $y = x^2 - 7x + 10$



c) $y = x^2 - 3x - 18$

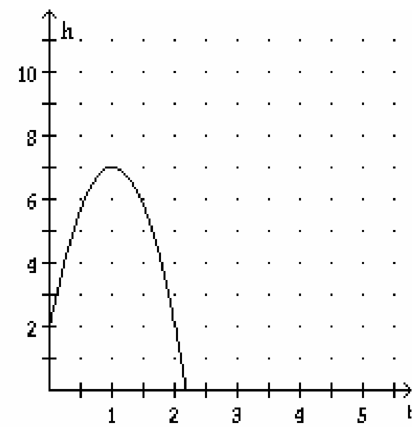


d) $y = x^2 - x - 30$



6.10.1 Problem Solving with Quadratic Graphs- Interpreting Parabolas

1. The graph below shows the height, in meters, of a diver jumping off a springboard versus time, in seconds.

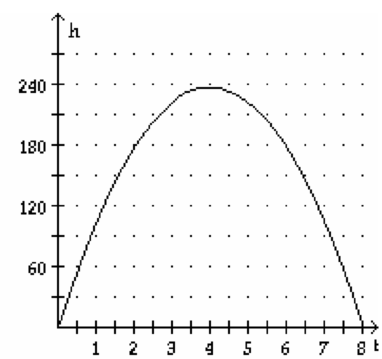


   a) What is the initial height of the jumper? _____
   What is this called in math terminology?

   b) Label and write the ordered pair of the vertex.
   What does this mean in real life?

   c) Label and write the ordered pairs of the roots/zeros. What do these points mean in real life?

2. The graph below shows the height of a toy rocket after it is launched.



   a) How many seconds is the rocket in the air?

   b) What math concept did you use to determine this?

   c) What is the maximum height of the rocket?

   d) At what time does the maximum height occur?

   e) At what times is the rocket 120 meters above the earth?

   f) Complete the table of values below with first and second differences.

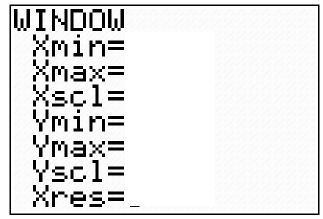
Time	Height	First Diff	Second Diff
0			
1			
2			
3			
4			

   g) What pattern did you observe in the second differences from the table? How does this prove that the relationship is quadratic?

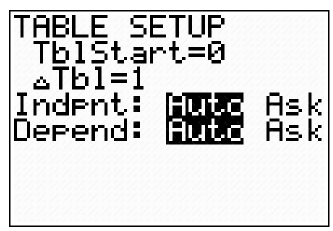
6.10.2 Applying Quadratic Relationships

There are many relationships that turn out to be quadratic. One of the most common is the relationship between the height of something (or someone) flying through the air and time.

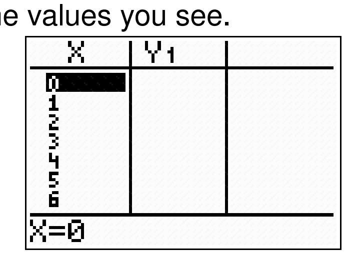
1. A football player kicks a ball of a football tee. The height of the ball, *h*, in metres after *t* seconds can be modelled using the formula: $h = -5t^2 + 20t$.
   a) Graph the relationship using your graphing calculator. Remember that you need to set your window settings. Record the window settings you used.
   b) Sketch your graph in the window at right. Make sure to label your axes.



   c) You can get the table of values for this relationship. Press $\boxed{\text{2nd}}$ and $\boxed{\text{WINDOW}}$ to access the table setup screen. Make sure your screen looks like the one given.



   d) Now press $\boxed{\text{2nd}}$ and $\boxed{\text{GRAPH}}$. Fill in the window with the values you see.



   e) For which times does the height not make sense? Why?

   f) What is the initial height of the ball? __________

   g) Where do you look in the table and the graph to determine the answer?

   h) Why does this make sense?

6.10.2 Applying Quadratic Relationships (continued)

i) What is the maximum height of the ball? __________

j) Where do you look in the table and the graph to determine the answer?

k) When does the ball hit the ground? __________

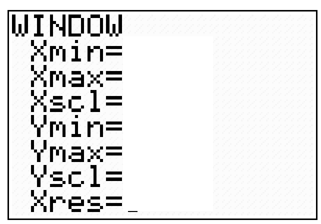
l) Where do you look in the table and the graph to determine the answer?

m) When is the ball more 10 m above the ground? (You may need to give approximate answers.)

Playing Football on Mars

2. The force of gravity on Mars is less than half that on Earth. A ball thrown upward can be modelled using $h = -2t^2 + 15t + 2$ where h is the height in m and t is the time in seconds.

a) Graph the relationship using your graphing calculator. Remember that you need to set your window settings. Record the window settings you used. You may need to play around with the settings until you see the full graph.



to

b) What is the initial height of the ball? __________

c) Explain what this means.

d) What is the maximum height of the ball? __________

e) When does the ball reach its maximum height? __________

f) When does the ball hit the ground? __________

g) When is the ball more 20 m above the ground? (You may need to give approximate answers.)

h) If the same ball was thrown upward on the Earth, how would you expect the relationship to change?

i) The force of gravity on Jupiter is much greater than on the Earth. If the same ball was thrown upward on Jupiter, how would you expect the relationship to change?

6.11.1 FRAME (Function Representation And Model Examples)

Algebraic Models	Tables of Values
Visual/Spatial/Concrete	Description/Key words
Contextual	Graphical Model

6.11.2 Review of Quadratic Relations

A) Linear vs. Quadratic Relations

- A linear relation forms a graph with a ______________.
- A quadratic relation forms a graph with shape of a _______________.
- Below is a table of value, determine if this relation is linear, quadratic or neither:

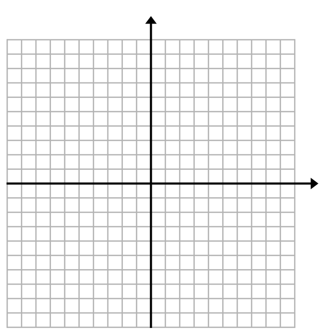
Time (x)	Distance (y)	First Differences	Second Difference

The relation is:

- A linear relation has the _____________equal and the _________________ equal to _____ _________.
- A quadratic relation has the _____________________ equal.

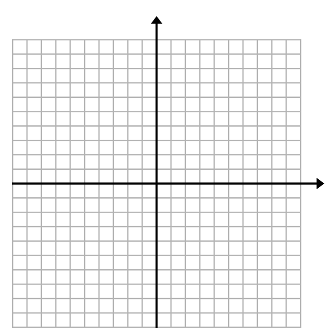
B) Features of the Graph of a Quadratic Relation

- A quadratic relation can be seen when a ball is thrown in the air and the height is measured versus time. A sketch of this graph might look:



This parabola is facing _________________
and the vertex is a ______________.

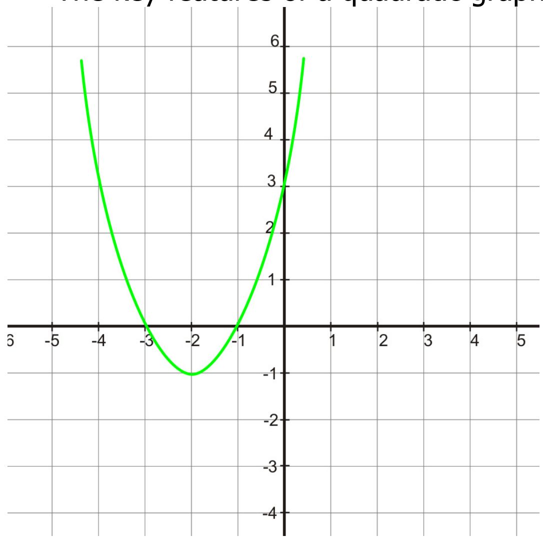
- A quadratic relation can be seen when a duck flies into the water, catches a fish and flies back out:



This parabola is facing _________________
and the vertex is a ______________.

6.11.2 Review of Quadratics (Continued)

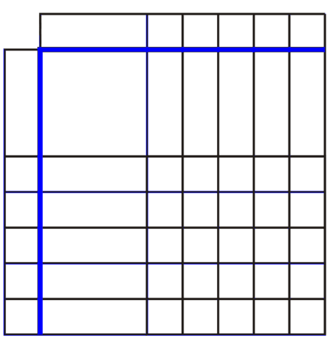
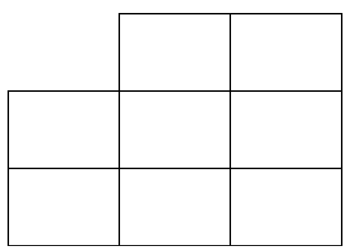
- The key features of a quadratic graph are:



C) Forms of a Quadratic Relation in Standard Form ($y = ax^2 + bx + c$)

- The y-intercept is the __________ or ____ term.
- We can change factored into standard form by _____________.
- There are three methods of expanding: _____________, _________________, and _______ _________
- Example 1: Find the y-intercept, x-intercepts of the following quadratic:

y = (x + 4)(x – 5)

Method 1: Tiles	Method 2: Table	Method 3: Algebra (FOIL)
		

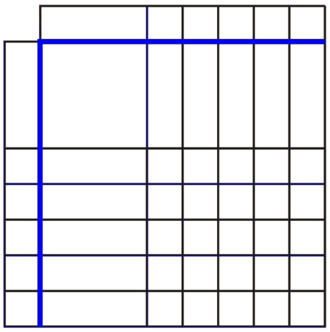
The y-intercept is: _____________________
The x-intercepts are: _______ and ______________

Example 2: Expand the following: **y = 2x(3x – 4)**

6.11.2 Review of Quadratics (Continued)

D) Forms of a Quadratic Relation in Factored Form: $y = (x - r)(x - s)$

- There are two methods of factoring: algebra tiles and algebra
- Example: factor $y = x^2 + 3x + 2$ using tiles



1. **Product/Sum Form: Factor $y = ax^2 + bx + c$**
   - Find the x-intercepts of the following:

$y = x^2 + 6x + 8$	$y = x^2 - 3x - 18$

2. **Common Factoring: $y = ax^2 + bx$**
   - Let us factor: $y = x^2 + 3x$
     This can be written as: ______________________________
     The factored form is y = ______________________________ or ______________________
   - Factor as state the x and y intercepts of:

$y = x^2 + 6x$	$y = 2x^2 - 2x - 60$

3. **Difference of Squares: $y = ax^2 - b^2$**
   - $y = x^2 - 4$ can be written as y = ______________________________
     factored form ________________________________

$y = x^2 - 16$	$y = 2x^2 - 200$